Comparing and Scaling
Ratio, Proportion, and Percent

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Two summers ago, a biologist captured, tagged, and released 20 puffins on an island. When she returned this past summer, she captured 50 puffins. Two of them were tagged. About how many puffins are on the island?

At camp, Miriam uses a pottery wheel to make 3 bowls in 2 hours. Duane makes 5 bowls in 3 hours. Who is the faster potter? Suppose they continue to work at the same pace. How long will it take each of them to make a set of 12 bowls?

It takes 100 maple trees to make 25 gallons of maple syrup. How many maple trees does it take for one gallon of syrup?
Many everyday problems and decisions call for comparisons. Which car is safer? Which horse is the fastest? Which Internet service is cheaper? In some cases, the comparisons involve only counting, measuring, or rating, then ordering the results from least to greatest. In other cases, more complex reasoning is required.

How would you answer the comparison questions on the previous page?

In this unit, you will explore many ways to compare numbers. You’ll learn how to both choose and use the best comparison strategies to solve problems and make decisions.
In *Comparing and Scaling*, you will develop several methods for comparing quantities. You will use these methods to solve problems.

You will learn how to

- Use informal language to ask comparison questions
  
  Examples:
  
  “What is the ratio of boys to girls in our class?”
  “What fraction of the class is going to the spring picnic?”
  “What percent of the girls play basketball?”
  “Which model of car has the best fuel economy?”

- Choose an appropriate method to make comparisons among quantities using ratios, percents, fractions, rates, or differences

- Find equivalent forms of given ratios and rates to scale comparisons up and down

- Find and interpret unit rates, and use them to make comparisons

- Use unit rates to write an equation to represent a pattern in a table of data

- Set up and solve proportions

- Use proportional reasoning to solve problems

As you work on the problems in this unit, ask yourself questions about problem situations that involve comparisons:

*What quantities should be compared?*

*What type of comparison will give the most useful information?*

*How can the comparison be expressed in different but useful ways?*

*How can given comparison data be used to make predictions about unknown quantities?*
Making Comparisons

Surveys may report people’s preferences in food, cars, or political candidates. Often, the favorites are easy to recognize. Explaining how much more popular one choice is than another can be more difficult. In this investigation, you will explore strategies for comparing numbers in accurate and useful ways. As you work on the problems, notice how the different ways of making comparisons send different messages about the numbers being compared.

1.1 Ads That Sell

An ad for the soft drink Bolda Cola starts like this:

Which Soft Drink Do You Like Better?

Bolda Cola

OR

COLA•NOLA

Take the Cola Taste Test Yourself!

To complete the ad, the Bolda Cola company plans to report the results of taste tests. A copywriter for the ad department has proposed four possible conclusions.
Problem 1.1 Exploring Ratios and Rates

A. Describe what you think each statement above means.

B. Which of the proposed statements do you think would be most effective in advertising Bolda Cola? Why?

C. Is it possible that all four statements are based on the same survey data? Explain your reasoning.

D. In what other ways can you express the claims in the four proposed advertising statements? Explain.

E. If you were to survey 1,000 cola drinkers, what numbers of Bolda Cola and Cola Nola drinkers would you expect? Explain.

Homework starts on page 10.

1.2 Targeting an Audience

Some middle and high school students earn money by delivering papers, mowing lawns, or baby-sitting. Students with money to spend are a target audience for some radio and television ads. Companies gather information about how much students watch television or listen to the radio. This information influences how they spend their advertising dollars.
As you work on this problem and the rest of the unit, you will see statements about ratio comparisons. In mathematics, it is acceptable to write ratios in different ways. Each way is useful.

It can be confusing to see a fraction representing a ratio. A ratio is usually, but not always, a part-to-part comparison. A fraction usually means a part-to-whole comparison. The context can help you decide whether a fraction represents a ratio.

**Problem 1.2 Analyzing Comparison Statements**

Students at Neilson Middle School are asked if they prefer watching television or listening to the radio. Of 150 students, 100 prefer television and 50 prefer radio.

A. How would you compare student preferences for radio or television?

B. Decide if each statement accurately reports results of the Neilson Middle School survey.

1. At Neilson Middle School, \(\frac{1}{3}\) of the students prefer radio to television.
2. Students prefer television to radio by a ratio of 2 to 1.
3. The ratio of students who prefer radio to television is 1 to 2.
4. The number of students who prefer television is 50 more than the number of students who prefer radio.
5. The number of students who prefer television is two times the number who prefer radio.
6. 50% of the students prefer radio to television.

C. Compare statements in parts (4) and (5) above. Which is more informative? Explain.

D. Consider only the accurate statements in Question B.

1. Which statement would best convince merchants to place ads on radio? Why?
2. Which statement would best convince merchants to place ads on television? Why?

ACE Homework starts on page 10.
**American Records**

People are amazed and amused by records like the highest mountain, the longest fingernails, or the most spoons balanced on a face. What you have learned so far can help you make comparisons. In Problem 1.3, you will compare the largest living trees of different species.

**Did You Know?**

The champion white “Wye” oak tree near Wye Mills, Maryland, was about 460 years old when it fell during a thunderstorm in 2002. When the tree fell, thousands came by to gawk, shed tears, and pick up a leaf or a twig. Maryland officials carefully gathered and stored as much of the tree as they could until a suitable use could be found.

The challenge to find a white oak bigger than the Wye Mills tree launched the National Register of Big Trees. The search led to the discovery of a new national champion white oak in Virginia.

You can describe the size of a tree by comparing it to other trees or familiar things.

**Selected Champion Trees**

<table>
<thead>
<tr>
<th>Tree Type</th>
<th>Circumference (ft)</th>
<th>Height (ft)</th>
<th>Spread/Diameter (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giant Sequoia (Calif.)</td>
<td>83.2</td>
<td>275</td>
<td>107</td>
</tr>
<tr>
<td>Coast Redwood (Calif.)</td>
<td>79.2</td>
<td>321</td>
<td>80</td>
</tr>
<tr>
<td>Swamp Chestnut Oak (Tenn.)</td>
<td>23.0</td>
<td>105</td>
<td>216</td>
</tr>
<tr>
<td>Florida Clossopetalum (Fla.)</td>
<td>0.4</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>White Oak (Md.)</td>
<td>31.8</td>
<td>96</td>
<td>119</td>
</tr>
</tbody>
</table>

**Source:** Washington Post
Problem 1.3 Writing Comparison Statements

A. Use the table on the previous page.

1. How many coast redwood spreads does it take to equal the spread of the white oak?

2. Kenning says that the spread of the white oak is greater than that of the coast redwood by a ratio of about 3 to 2. Is he correct? Explain.

3. Mary says the difference between the heights of the coast redwood and the giant sequoia is 46 feet. Is she correct? Explain.

4. How many giant sequoia spreads does it take to equal the spread of the swamp chestnut oak?

5. Jaime says the spread of the giant sequoia is less than 50% of the spread of the swamp chestnut oak. Is he correct?

6. Len says the circumference of the swamp chestnut oak is about three fourths the circumference of the white oak. Is he correct?

B. The tallest person in history, according to the Guinness Book of World Records, was Robert Wadlow. He was nearly 9 feet tall. Write two statements comparing Wadlow to the trees in the table. Use fractions, ratios, percents, or differences.

C. Average waist, height, and arm-span measurements for a small group of adult men are given.

<table>
<thead>
<tr>
<th>Waist</th>
<th>Height</th>
<th>Arm Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 inches</td>
<td>72 inches</td>
<td>73 inches</td>
</tr>
</tbody>
</table>

Write two statements comparing the data on these men to the trees in the table. Use fractions, ratios, percents, or differences.

D. When a problem requires comparison of counts or measurements, how do you decide whether to use differences, ratios, fractions, or percents?

ACE Homework starts on page 10.
Applications

1. In a comparison taste test of two drinks, 780 students preferred Berry Blast. Only 220 students preferred Melon Splash. Complete each statement.
   a. There were □ more people who preferred Berry Blast.
   b. In the taste test, □% of the people preferred Berry Blast.
   c. People who preferred Berry Blast outnumbered those who preferred Melon Splash by a ratio of □ to □.

2. In a comparison taste test of new ice creams invented at Moo University, 750 freshmen preferred Cranberry Bog ice cream while 1,250 freshmen preferred Coconut Orange ice cream.

   Complete each statement.
   a. The fraction of freshmen who preferred Cranberry Bog is □.
   b. The percent of freshmen who preferred Coconut Orange is □%.
   c. Freshmen who preferred Coconut Orange outnumbered those who preferred Cranberry Bog by a ratio of □ to □.

3. A town considers whether to put in curbs along the streets. The ratio of people who support putting in curbs to those who oppose it is 2 to 5.

   a. What fraction of the people oppose putting in curbs?
   b. If 210 people in the town are surveyed, how many do you expect to favor putting in curbs?
   c. What percent of the people oppose putting in curbs?
Students at a middle school are asked to record how they spend their time from midnight on Friday to midnight on Sunday. Carlos records his data in the table below. Use the table for Exercises 4–7.

**Weekend Activities**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleeping</td>
<td>18</td>
</tr>
<tr>
<td>Eating</td>
<td>2.5</td>
</tr>
<tr>
<td>Recreation</td>
<td>8</td>
</tr>
<tr>
<td>Talking on the Phone</td>
<td>2</td>
</tr>
<tr>
<td>Watching Television</td>
<td>6</td>
</tr>
<tr>
<td>Doing Chores or Homework</td>
<td>2</td>
</tr>
<tr>
<td>Other</td>
<td>9.5</td>
</tr>
</tbody>
</table>

4. How would you compare how Carlos spent his time on various activities over the weekend? Explain.

5. Decide if each statement is an accurate description of how Carlos spent his time that weekend.

   a. He spent one sixth of his time watching television.
   b. The ratio of hours spent watching television to hours spent doing chores or homework is 3 to 1.
   c. Recreation, talking on the phone, and watching television took about 33% of his time.
   d. Time spent doing chores or homework was only 20% of the time spent watching television.
   e. Sleeping, eating, and “other” activities took up 12 hours more than all other activities combined.

6. Estimate what the numbers of hours would be in your weekend activity table. Then write a ratio statement like statement (b) to fit your data.

7. Write other accurate statements comparing Carlos’s use of weekend time for various activities. Use each concept at least once.

   a. ratio
   b. difference
   c. fraction
   d. percent
8. A class at Middlebury Middle School collected data on the kinds of movies students prefer. Complete each statement using the table.

**Types of Movies Preferred by Middlebury Students**

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Seventh-Graders</th>
<th>Eighth-Graders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>75</td>
<td>90</td>
</tr>
<tr>
<td>Comedy</td>
<td>105</td>
<td>150</td>
</tr>
<tr>
<td>Total</td>
<td>180</td>
<td>240</td>
</tr>
</tbody>
</table>

a. The ratio of seventh-graders who prefer comedies to eighth-graders who prefer comedies is \( \underline{75} \) to \( \underline{90} \).

b. The fraction of total students (both seventh- and eighth-graders) who prefer action movies is \( \underline{\frac{75}{180}} \).

c. The fraction of seventh-graders who prefer action movies is \( \underline{\frac{75}{180}} \).

d. The percent of total students who prefer comedies is \( \underline{\frac{75}{180} \times 100} \% \).

e. The percent of eighth-graders who prefer action movies is \( \underline{\frac{90}{240} \times 100} \% \).

f. Grade \( \underline{7} \) has the greater percent of students who prefer action movies.

9. Use the table.

**Selected Champion Trees**

<table>
<thead>
<tr>
<th>Tree Type</th>
<th>Height (ft)</th>
<th>Spread (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Florida Crossopetalum</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>White Oak</td>
<td>96</td>
<td>119</td>
</tr>
</tbody>
</table>

a. The height of the crossopetalum (kroh soh PET uh lum) is what fraction of the height of the white oak?

b. The height of the crossopetalum is what percent of the height of the white oak?

c. The spread of the crossopetalum is what fraction of the spread of the white oak?

d. The spread of the crossopetalum is what percent of the spread of the white oak?

10. In a survey, 100 students were asked if they prefer watching television or listening to the radio. The results show that 60 students prefer watching television while 40 prefer listening to the radio. Use each concept at least once to express student preferences.

a. ratio

b. percent

c. fraction

d. difference
Connections

11. A fruit bar is 5 inches long. The bar will be split into two pieces. For each situation, find the lengths of the two pieces.
   a. One piece is \( \frac{3}{10} \) of the whole bar.
   b. One piece is 60% of the bar.
   c. One piece is 1 inch longer than the other.

12. Exercise 11 includes several numbers or quantities: 5 inches, 3, 10, 60%, and 1 inch. Determine whether each number or quantity refers to the whole, a part, or the difference between two parts.

The sketches below show two members of the Grump family. The figures are geometrically similar. Use the figures for Exercises 13–16.

13. Write statements comparing the lengths of corresponding segments in the two Grump drawings. Use each concept at least once.
   a. ratio
   b. fraction
   c. percent
   d. scale factor

14. Write statements comparing the areas of the two Grump drawings. Use each concept at least once.
   a. ratio
   b. fraction
   c. percent
   d. scale factor

15. How long is the segment in the smaller Grump that corresponds to the 1.4-inch segment in the larger Grump?

16. Multiple Choice The mouth of the smaller Grump is 0.6 inches wide. How wide is the mouth of the larger Grump?
   A. 0.4 in.    B. 0.9 in.    C. 1 in.    D. 1.2 in.
The drawing below shows the Big Wheel spinner used in a game at the Waverly School Fun Night. It costs 20 cents to spin the wheel, and winners receive $1.00. The chart shows the data from 236 spins of the Big Wheel. Use the spinner and the chart for Exercises 17–21.

17. The sectors of the spinner are identical in size. What is the measure in degrees of each central angle?

18. You play the game once. What is the theoretical probability that you win?

19. Do the results in the table agree with the probability statement you made in Exercise 18? Why or why not?

20. Write statements comparing the number of wins to the number of losses. Use each concept at least once.
   a. ratio  
   b. percent  
   c. difference

21. Which comparison from Exercise 20 is the best way to convey probability information about this game? Explain.

22. Copy the number line below. Add labels for 0.25, \( \frac{6}{8} \), 1.75, and 1.3.

23. Write two unequal fractions with different denominators. Which fraction is greater? Explain.

24. Write a fraction and a decimal so that the fraction is greater than the decimal. Explain.
Copy each pair of numbers in Exercises 25–33. Insert <, >, or = to make a true statement.

25. \( \frac{4}{5} \) \( \leq \) \( \frac{11}{12} \)
26. \( \frac{14}{21} \) \( \leq \) \( \frac{10}{15} \)
27. \( \frac{7}{9} \) \( \leq \) \( \frac{3}{4} \)
28. 2.5 \( \leq \) 0.259
29. 30.17 \( \leq \) 30.018
30. 0.006 \( \leq \) 0.0060
31. 0.45 \( \leq \) \( \frac{9}{20} \)
32. \( \frac{3}{4} \) \( \leq \) 1.5
33. \( \frac{1}{4} \) \( \leq \) 1.3

**Extensions**

34. Rewrite this ad so that it will be more effective.

Three thousand out of four thousand five hundred dentists surveyed recommend sugarless gum to their patients who chew gum.

35. Use the table below.

<table>
<thead>
<tr>
<th>Where Food Is Eaten</th>
<th>1990</th>
<th>1998</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>$303,900,000,000</td>
<td>$401,800,000,000</td>
</tr>
<tr>
<td>Away From Home</td>
<td>$168,800,000,000</td>
<td>$354,400,000,000</td>
</tr>
</tbody>
</table>

**SOURCE:** U.S. Census Bureau. Go to PHSchool.com for a data update. Web Code: ang-9041

a. Compare money spent on food eaten at home and food eaten away from home to the total money spent for food. Write statements for each year.

b. Explain how the statements you wrote in part (a) show the money spent for food away from home increasing or decreasing in relation to the total spent for food.
Use the table for Exercises 36–41.

<table>
<thead>
<tr>
<th>Placement</th>
<th>1990</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newspapers</td>
<td>$32,281</td>
<td>$46,582</td>
</tr>
<tr>
<td>Magazines</td>
<td>$6,803</td>
<td>$11,096</td>
</tr>
<tr>
<td>Television</td>
<td>$29,073</td>
<td>$50,843</td>
</tr>
<tr>
<td>Radio</td>
<td>$8,726</td>
<td>$16,930</td>
</tr>
<tr>
<td>Yellow Pages</td>
<td>$8,926</td>
<td>$12,666</td>
</tr>
<tr>
<td>Internet</td>
<td>$0</td>
<td>$1,840</td>
</tr>
<tr>
<td>Direct Mail</td>
<td>$23,370</td>
<td>$41,601</td>
</tr>
<tr>
<td>Other</td>
<td>$20,411</td>
<td>$33,671</td>
</tr>
<tr>
<td>Total</td>
<td>$129,590</td>
<td>$215,229</td>
</tr>
</tbody>
</table>

36. Which placement has the greatest difference in advertising dollars between 1990 and 2000?

37. Find the percent of all advertising dollars spent on each placement in 1990.

38. Find the percent of all advertising dollars spent on each placement in 2000.


40. Suppose you were thinking about investing in either a television station or a radio station. Which method of comparing advertising costs (differences or percents) makes television seem like the better investment? Which makes radio seem like the better investment?

41. Suppose you are a reporter writing an article about trends in advertising over time. Which method of comparison would you choose?
In this investigation, you explored several ways of comparing numbers. The problems were designed to help you understand and use different comparison strategies and recognize when each is most useful. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain what you think each word means when it is used to make a comparison.
   a. ratio
   b. percent
   c. fraction
   d. difference

2. Give an example of a situation using each concept to compare two quantities.
   a. ratio
   b. percent
   c. fraction
   d. difference
Comparing Ratios, Percents, and Fractions

You used ratios, fractions, percents, and differences to compare quantities in Investigation 1. Now, you will develop strategies for choosing and using an appropriate comparison strategy. As you work through the problems, you will make sense of the statements in the Did You Know?

Did You Know?

- In 2001, 20.8% of all radio stations in the United States had country music as their primary format, while only 4.5% had a Top-40 format.
- For the first 60 miles of depth, the temperature of Earth increases 1°F for every 100 to 200 feet.
- In 2000, cancer accounted for about $\frac{1}{3}$ of all deaths in the United States.
- In 2001, silver compact cars and silver sports cars outsold black cars by a ratio of 5 to 3.

For: Information about any of these topics
Web Code: ane-9031
Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.

One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange-juice concentrate. To find the mix that tastes best, they decide to test some mixes.

**Mix A**
- 2 cups concentrate
- 3 cups cold water

**Mix B**
- 5 cups concentrate
- 9 cups cold water

**Mix C**
- 1 cup concentrate
- 2 cups cold water

**Mix D**
- 3 cups concentrate
- 5 cups cold water

### Problem 2.1 Developing Comparison Strategies

A. Which mix will make juice that is the most “orangey”? Explain.
B. Which mix will make juice that is the least “orangey”? Explain.
C. Which comparison statement is correct? Explain.
   
   \[
   \frac{5}{9} \text{ of Mix B is concentrate.} \\
   \frac{5}{14} \text{ of Mix B is concentrate.}
   \]
D. Assume that each camper will get \(\frac{1}{2}\) cup of juice.
   1. For each mix, how many batches are needed to make juice for 240 campers?
   2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?
E. For each mix, how much concentrate and how much water are needed to make 1 cup of juice?

*Homework starts on page 24.*
The camp dining room has two kinds of tables. A large table seats ten people. A small table seats eight people. On pizza night, the students serving dinner put four pizzas on each large table and three pizzas on each small table.

Problem 2.2 More Comparison Strategies

A. Suppose the pizzas are shared equally by everyone at the table. Does a person sitting at a small table get the same amount as a person sitting at a large table? Explain your reasoning.

B. Which table relates to $\frac{3}{8}$? What do the 3 and the 8 mean? Is $\frac{3}{8}$ a part-to-whole comparison or a part-to-part comparison?

C. Selena thinks she can decide at which table a person gets the most pizza. She uses the following reasoning:

$$10 - 4 = 6 \quad \text{and} \quad 8 - 3 = 5$$

so the large table is better.

1. What does the 6 mean and what does the 5 mean in Selena’s method of reasoning?

2. Do you agree or disagree with Selena’s method?

3. Suppose you put nine pizzas on the large table. What answer does Selena’s method give? Does this answer make sense?

4. What can you now say about Selena’s method?
D. 1. The ratio of large tables to small tables in the dining room is 8 to 5. There are exactly enough seats for the 240 campers. How many tables of each kind are there?

2. What fraction of the campers sit at small tables?

3. What percent of the campers sit at large tables?

ACE Homework starts on page 24.

2.3 Finding Equivalent Ratios

It is often helpful, when forming ratios, to replace the actual numbers being compared with simpler numbers that have the same relationship to each other.

• People prefer Bolda Cola over Cola Nola by a ratio of 17,139 to 11,426, or 3 to 2.

• Students prefer television to radio by a ratio of 100 to 50, or 2 to 1.

• Monthly sales of Reader’s Digest magazine exceed those of National Geographic by 11,044,694 to 6,602,650, or about 3 to 2.

Getting Ready for Problem 2.3

Suppose all classes at your grade level took the cola taste test. The result was 100 to 80 in favor of Bolda Cola.

• How do you scale down this ratio to make it easier to understand?

• What are some other ratios equivalent to this ratio in which the numbers are greater? Finding greater numbers is scaling up the ratio.

• How is scaling ratios like finding equivalent fractions for \( \frac{100}{80} \)? How is it different?
Problem 2.3 Scaling Ratios

One of Ming’s tasks at the county zoo’s primate house is to mix food for the chimpanzees. The combination of high-fiber nuggets and high-protein nuggets changes as the chimps grow from babies to adults.

Ming has formulas for mixing high-fiber and high-protein nuggets for the chimps.

- Baby chimps: 2 cups high-fiber nuggets and 3 cups high-protein nuggets per serving
- Young adult chimps: 6 cups high-fiber nuggets and 4 cups high-protein nuggets per serving
- Older chimps: 4 cups high-fiber nuggets and 2 cups high-protein nuggets per serving

A. 1. What amounts of high-fiber and high-protein nuggets will Ming need when she has to feed 2 baby chimps? 3 baby chimps? 4 baby chimps?

Copy and complete the table below.

<table>
<thead>
<tr>
<th>Number of Baby Chimps</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of High-Fiber Nuggets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cups of High-Protein Nuggets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What patterns do you see in your table?

3. Ming puts 48 cups of high-protein nuggets into the baby chimp mix. How many cups of high-fiber nuggets does she put into the mix? Explain.

4. Ming has a total of 125 cups of mix for baby chimps. How many cups of high-fiber nuggets are in the mix? Explain.

B. 1. What is the ratio of high-fiber to high-protein nuggets for young adult chimps?

2. Scale this ratio up to show the ratio of high-fiber to high-protein nuggets that will feed 21 young adult chimps.

3. To feed 18 young adults, you need 108 cups of high-fiber nuggets and 72 cups of high-protein nuggets. Show how to scale down this ratio to feed 3 young adult chimps.
C. 1. Darla wants to compare the amount of high-fiber nuggets to the total amount of food mix for young adult chimps. She makes this claim:

“High-fiber nuggets are \( \frac{3}{2} \) of the total.”

Lamar says Darla is wrong. He makes this claim:

“High-fiber nuggets are \( \frac{3}{3} \) of the total.”

Who is correct? Explain.

2. What fraction of the total amount of food mix for older chimps is high-fiber nuggets?

3. Suppose the ratio of male chimps to female chimps in a zoo is 5 to 4. What fraction of the chimps are male?

4. Suppose \( \frac{2}{3} \) of the chimps in a zoo are female. Find the ratio of female chimps to male chimps in that zoo.
Applications

As you work on the ACE exercises, try a variety of reasoning methods. Then think about conditions when each method seems most helpful.

1. Compare these four mixes for apple juice.

<table>
<thead>
<tr>
<th>Mix</th>
<th>Concentrate</th>
<th>Cold Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>5 cups</td>
<td>8 cups</td>
</tr>
<tr>
<td>X</td>
<td>3 cups</td>
<td>6 cups</td>
</tr>
<tr>
<td>Y</td>
<td>6 cups</td>
<td>9 cups</td>
</tr>
<tr>
<td>Z</td>
<td>3 cups</td>
<td>5 cups</td>
</tr>
</tbody>
</table>

a. Which mix would make the most “appley” juice?
b. Suppose you make a single batch of each mix. What fraction of each batch is concentrate?
c. Rewrite your answers to part (b) as percents.
d. Suppose you make only 1 cup of Mix W. How much water and how much concentrate do you need?

2. Examine these statements about the apple juice mixes in Exercise 1. Decide whether each is accurate. Give reasons for your answers.

a. Mix Y has the most water, so it will taste least “appley.”
b. Mix Z is the most “appley” because the difference between the concentrate and water is 2 cups. It is 3 cups for each of the others.
c. Mix Y is the most “appley” because it has only 1 \( \frac{1}{2} \) cups of water for each cup of concentrate. The others have more water per cup.
d. Mix X and Mix Y taste the same because you just add 3 cups of concentrate and 3 cups of water to turn Mix X into Mix Y.
3. If possible, change each comparison of concentrate to water into a ratio. If not possible, explain why.
   a. The mix is 60% concentrate.
   b. The fraction of the mix that is water is \(\frac{3}{5}\).
   c. The difference between the amount of concentrate and water is 4 cups.

4. At camp, Miriam uses a pottery wheel to make three bowls in 2 hours. Duane makes five bowls in 3 hours.
   a. Who makes bowls faster, Miriam or Duane?
   b. At the same pace, how long will it take Miriam to make a set of 12 bowls?
   c. At the same pace, how long will it take Duane to make a set of 12 bowls?

5. Guests at a pizza party are seated at 3 tables. The small table has 5 seats and 2 pizzas. The medium table has 7 seats and 3 pizzas. The large table has 12 seats and 5 pizzas. The pizzas at each table are shared equally. At which table does a guest get the most pizza?

6. For each business day, news reports tell the number of stocks that gained (went up in price) and the number that declined (went down in price). In each of the following pairs of reports, determine which is better news for investors.
   a. [Gains outnumber declines by a ratio of 5 to 3.] OR [Gains outnumber declines by a ratio of 7 to 5.]
   b. [Gains outnumber declines by a ratio of 9 to 5.] OR [Gains outnumber declines by a ratio of 6 to 3.]
   c. [Declines outnumber gains by a ratio of 10 to 7.] OR [Declines outnumber gains by a ratio of 6 to 4.]
7. Suppose a news story about the Super Bowl claims “Men outnumbered women in the stadium by a ratio of 9 to 5.” Does this mean that there were 14 people in the stadium—9 men and 5 women? If not, what does the statement mean?

8. **Multiple Choice** Which of the following is a correct interpretation of the statement “Men outnumbered women by a ratio of 9 to 5?”
   A. There were four more men than women.
   B. The number of men was 1.8 times the number of women.
   C. The number of men divided by the number of women was equal to the quotient of $5 \div 9$.
   D. In the stadium, five out of nine fans were women.

**Connections**

9. If possible, change each comparison of red paint to white paint to a percent comparison. If it is not possible, explain why.
   a. The fraction of a mix that is red paint is $\frac{1}{4}$.
   b. The ratio of red to white paint in a different mix is 2 to 5.

10. If possible, change each comparison to a fraction comparison. If it is not possible, explain why.
   a. The nut mix has 30% peanuts.
   b. The ratio of almonds to other nuts in the mix is 1 to 7.

11. Find a value that makes each sentence correct.
   a. $\frac{3}{15} = \frac{\phantom{30}}{30}$
   b. $\frac{\frac{1}{2}}{20} \leq \frac{\phantom{30}}{20}$
   c. $\frac{\phantom{20}}{20} \geq \frac{3}{5}$
   d. $\frac{9}{30} \leq \frac{\phantom{15}}{15}$
   e. $\frac{\phantom{12}}{12} \geq \frac{3}{4}$
   f. $\frac{9}{21} = \frac{\phantom{12}}{\phantom{11}}$
12. Use the table to answer parts (a)–(e).

<table>
<thead>
<tr>
<th></th>
<th>Ages 12–17</th>
<th>Ages 55–64</th>
</tr>
</thead>
<tbody>
<tr>
<td>People Who Walk</td>
<td>3,781,000</td>
<td>8,694,000</td>
</tr>
<tr>
<td>Total in Group</td>
<td>23,241,000</td>
<td>22,662,000</td>
</tr>
</tbody>
</table>

**Participation in Walking for Exercise**

a. What percent of the 55–64 age group walk for exercise?

b. What percent of the 12–17 age group walk for exercise?

c. Write a ratio statement to compare the number of 12- to 17-year-olds who walk to the number of 55- to 64-year-olds who walk. Use approximate numbers to simplify the ratio.

d. Write a ratio statement to compare the percent of 12- to 17-year-olds who walk for exercise to the percent of 55- to 64-year-olds who walk for exercise.

e. Which data—actual numbers of walkers or percents—would you use in comparing the popularity of exercise walking among various groups? Explain.

13. The probability of getting a sum of 5 when you roll two number cubes is \( \frac{4}{36} \). How many times should you expect to get a sum of 5 if you roll the cubes each number of times?

a. 9  
   b. 18  
   c. 27  
   d. 100  
   e. 450

14. For each diagram, write three statements comparing the areas of the shaded and unshaded regions. In one statement, use fraction ideas to express the comparison. In the second, use percent ideas. In the third, use ratio ideas.

![Diagram a](image)

![Diagram b](image)

15. **Multiple Choice** Choose the value that makes \( \frac{18}{30} = \frac{\square}{15} \) correct.

F. 7  
   G. 8  
   H. 9  
   J. 10

16. **Multiple Choice** Choose the value that makes \( \frac{\square}{15} \leq \frac{3}{5} \) correct.

A. 9  
   B. 10  
   C. 11  
   D. 12

**Investigation 2** Comparing Ratios, Percents, and Fractions
17. Find a value that makes each sentence correct. Explain your reasoning in each case.

\[ \frac{3}{4} = \frac{\text{?}}{12} \quad \text{b.} \quad \frac{3}{4} < \frac{\text{?}}{12} \quad \text{c.} \quad \frac{3}{4} > \frac{\text{?}}{12} \quad \text{d.} \quad \frac{9}{12} = \frac{\text{?}}{\text{?}} \]

18. The sketches show floor plans for dorm rooms for two students and for one student.

a. Are the floor plans similar rectangles? If so, what is the scale factor? If not, why not?

b. What is the ratio of floor areas of the two rooms (including space under the beds and desks)?

c. Which type of room gives more space per student?

19. Find values that make each sentence correct.

\[ \frac{6}{14} = \frac{\text{?}}{21} = \frac{\text{?}}{28} \quad \text{b.} \quad \frac{27}{36} = \frac{8}{\text{?}} = \frac{\text{?}}{63} \quad \text{c.} \quad \frac{6}{20} = \frac{\text{?}}{25} = \frac{\text{?}}{30} \quad \text{d.} \quad \frac{\text{?}}{8} = \frac{15}{32} = \frac{24}{\text{?}} \]

20. Suppose a news story reports, “90% of the people in the Super Bowl stadium were between the ages of 25 and 55.” Alicia thinks this means only 100 people were in the stadium, and 90 of them were between 25 and 55 years of age. Do you agree with her? If not, what does the statement mean?

21. Suppose a news story reports, “A survey found that \( \frac{4}{7} \) of all Americans watched the Super Bowl on television.” Bishnu thinks this means the survey reached seven people and four of them watched the Super Bowl on television. Do you agree with him? If not, what does the statement mean?
Extensions

22. Mammals vary in the length of their pregnancies, or gestations. *Gestation* is the time from conception to birth. Use the table to answer the questions that follow.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Gestation (days)</th>
<th>Life Span (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chipmunk</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>Cat</td>
<td>63</td>
<td>12</td>
</tr>
<tr>
<td>Fox</td>
<td>52</td>
<td>7</td>
</tr>
<tr>
<td>Lion</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>Black Bear</td>
<td>219</td>
<td>18</td>
</tr>
<tr>
<td>Gorilla</td>
<td>258</td>
<td>20</td>
</tr>
<tr>
<td>Moose</td>
<td>240</td>
<td>12</td>
</tr>
<tr>
<td>Giraffe</td>
<td>425</td>
<td>10</td>
</tr>
<tr>
<td>Elephant (African)</td>
<td>660</td>
<td>35</td>
</tr>
</tbody>
</table>

*a. Plan a way to compare life span and gestation time for animals and use it with the data.*

*b. Which animal has the greatest ratio of life span to gestation time? Which has the least ratio?*

*c. Plot the data on a coordinate graph using \((\text{gestation, life span})\) as data points. Describe any interesting patterns that you see. Decide whether there is any relation between the two variables. Explain how you reached your conclusion.*

*d. What pattern would you expect to see in a graph if each statement were true?*

   *i. Longer gestation time implies longer life span.*

   *ii. Longer gestation time implies shorter life span.*
23. The city of Spartanville runs two summer camps—the Green Center and the Blue Center. The table below shows recent attendance at the two camps.

<table>
<thead>
<tr>
<th></th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>125</td>
<td>70</td>
</tr>
<tr>
<td>Girls</td>
<td>75</td>
<td>30</td>
</tr>
</tbody>
</table>

In this exercise, you will show how several approaches can be used to answer the following question.

Which center seems to offer a camping program that appeals best to girls?

a. What conclusion would you draw if you focused on the differences between the numbers of boy and girl campers from each center?

b. How could you use fractions to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?

c. How could you use percents to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?

d. How could you use ratios to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?
24. Use the table below.

<table>
<thead>
<tr>
<th>Sport</th>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basketball</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>Football</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Soccer</td>
<td>120</td>
<td>85</td>
</tr>
<tr>
<td><strong>Total Surveyed</strong></td>
<td><strong>160</strong></td>
<td><strong>225</strong></td>
</tr>
</tbody>
</table>

**Participation in Team Sports at Springbrook Middle School**

a. In which sport do boys most outnumber girls?

b. In which sport do girls most outnumber boys?

c. The participation in these team sports is about the same for students at Key Middle School.
   i. Suppose 250 boys at Key play sports. How many would you expect to play each of the three sports?
   ii. Suppose 240 girls at Key play sports. How many would you expect to play each of the three sports?
In this investigation, you solved problems by comparing ratios, percents, and fractions. You also used ratio, percent, and fraction data to solve problems of larger or smaller scale. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. The director of a recreation center wants to compare the 10 boys to the 20 girls who attend its camping program.
   a. How would you make a comparison using fractions?
   b. How would you make a comparison using percents?
   c. How would you make a comparison using ratios?
   d. How is your percent comparison related to your ratio comparison?
   e. How is your fraction comparison related to your percent comparison?

2. a. Explain how you would scale up the ratio 10 boys to 14 girls to find equivalent ratios.
   b. Explain how you would scale down the ratio 10 boys to 14 girls to find equivalent ratios.
Comparing and Scaling Rates

The following examples illustrate situations involving another strategy to compare numbers.

- My mom’s car gets 45 miles per gallon on the expressway.
- We need two sandwiches for each person at the picnic.
- I earn $3.50 per hour baby-sitting for my neighbor.
- The mystery meat label says 355 Calories per 6-ounce serving.
- My brother’s top running rate is 8.5 kilometers per hour.

Each of these statements compares two different quantities. For example, one compares miles to gallons of gas. A comparison of two quantities measured in different units is a **rate**. You have used rates in earlier problems. For example, you used rates in finding pizza per person.

**Getting Ready for Problem 3.1**

- What two quantities are being compared in the rate statements above?
- Which of the rate statements is different from the others?
Stores, catalogs, and Web sites often use rates in their ads. The ads sometimes give the costs for several items. You might see an offer like the one shown at the right.

The listed prices are for orders of 10, 15, or 20 calculators. But it’s possible to figure the price for any number you want to purchase. One way to figure those prices is to build a rate table. A rate table is started below.

<table>
<thead>
<tr>
<th>Price of Calculators for Schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Purchased</td>
</tr>
<tr>
<td>Fraction</td>
</tr>
<tr>
<td>Scientific</td>
</tr>
<tr>
<td>Graphing</td>
</tr>
</tbody>
</table>

### Problem 3.1 Making and Using a Rate Table

Suppose you take orders over the phone for the calculator company. You should be quick with price quotes for orders of different sizes.

**A.** Build a rate table like the one above. Fill in prices for each type of calculator for orders of the sizes shown.

Use your rate table to answer Questions B–F.

**B.** How much does it cost to buy 53 fraction calculators? How much to buy 27 scientific calculators? How much to buy 9 graphing calculators?

**C.** How many fraction calculators can a school buy if it can spend $390? What if the school can spend only $84?

**D.** How many graphing calculators can a school buy if it can spend $2,500? What if the school can spend only $560?

**E.** What arithmetic operation (addition, subtraction, multiplication or division) do you use to find the cost per calculator?

**F.** Write an equation for each kind of calculator to show how to find the price for any number ordered.

---

ACE Homework starts on page 40.
Sascha cycled on a route with different kinds of conditions. Sometimes he went uphill, sometimes he went mostly downhill. Sometimes he was on flat ground. He stopped three times to record his time and distance:

- Stop 1: 5 miles in 20 minutes
- Stop 2: 8 miles in 24 minutes
- Stop 3: 15 miles in 40 minutes

**Problem 3.2 Finding Rates**

Show your work. Label any rate that you find with appropriate units.

A. Find Sascha’s rate in miles per hour for each part of the route.

B. 1. On which part was Sascha cycling fastest? On which part was he cycling slowest?
   2. How do your calculations in Question A support your answers?

C. Suppose you can maintain a steady rate of 13 miles per hour on a bike. How long will it take you to travel the same distance Sascha traveled in 1 hour and 24 minutes?

D. Suppose you were racing Sascha. What steady rate would you have to maintain to tie him?

**ACE** Homework starts on page 40.

**Did You Know?**

The highest rate ever recorded on a pedal-powered bicycle was 166.944 miles per hour. Fred Rompelberg performed this amazing feat on October 3, 1995, at the Bonneville Salt Flats in Utah. He was able to reach this rate by following a vehicle. The vehicle acted as a windshield for him and his bicycle.
Comparing CD Prices

The ads below use rates to describe sale prices. To compare prices in sales such as these, it’s often useful to find a unit rate. A **unit rate** is a rate in which one of the numbers being compared is 1 unit. The comparisons “45 miles per gallon,” “$3.50 per hour,” “8.5 kilometers per hour,” and “two sandwiches for each person” are all unit rates. “Per gallon” means “for one gallon” and “per hour” means “for one hour.”

**Problem 3.3 Unit Rates and Equations**

Use unit rates to compare the ad prices and to find the costs of various numbers of CDs at each store.

**A.** Which store has the lower price per CD?

**B.** For each store, write an equation (a rule) that you can use to calculate the cost \( c \) for any purchase of \( n \) compact discs.

**C.** Use the equations you just wrote for Question B. Write new equations to include 5% sales tax on any purchase.
D. Suppose a Web site sells CDs for $8.99 per disc. There is no tax, but there is a shipping charge of $5 for any order. Write an equation to give the cost \( c \) of any order for \( n \) discs from the Web site.

E. Use your equations from Question C or make a rate table to answer each question.
   1. How many discs do you have to order from the Web site to get a better deal than buying from Music City?
   2. How many discs do you have to order from the Web site to get a better deal than buying from CD World?

ACE Homework starts on page 40.

3.4 What Does Dividing Tell You?

In this problem, the questions will help you decide which way to divide when you are finding a unit rate. The questions will also help you with the meaning of the quotient after you divide.

Getting Ready for Problem 3.4

Dario has two options for buying boxes of pasta. At CornerMarket he can buy seven boxes of pasta for $6. At SuperFoodz he can buy six boxes of pasta for $5.

At CornerMarket, he divided 7 by 6 and got 1.16666667. He then divided 6 by 7 and got 0.85714286. He was confused. What do these numbers tell about the price of boxes of pasta at CornerMarket?

Decide which makes more sense to you. Use that division strategy to compare the two store prices. Which store offers the better deal?
Problem 3.4 Two Different Rates

Use division to find unit rates to solve the following questions. Label each unit rate.

A. SuperFoodz has oranges on sale at 10 for $2.
   1. What is the cost per orange?
   2. How many oranges can you buy for $1?
   3. What division did you perform in each case? How did you decide what each division means?
   4. Complete this rate table to show what you know.

<table>
<thead>
<tr>
<th>Oranges</th>
<th>10</th>
<th></th>
<th>1</th>
<th>20</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2.00</td>
<td>$1.00</td>
<td>$2.00</td>
<td>$2.00</td>
<td>$2.60</td>
</tr>
</tbody>
</table>

B. Noralie used 22 gallons of gas to go 682 miles.
   1. What are the two unit rates that she might compute?
   2. Compute each unit rate and tell what it means.
   3. Which seems more useful to you? Why?
C. It takes 100 maple trees to make 25 gallons of maple syrup.
   1. How many maple trees does it take for 1 gallon of syrup?
   2. How much syrup can you get from one maple tree?

D. A 5-minute shower requires about 18 gallons of water.
   1. How much water per minute does a shower take?
   2. How long does a shower last if you use only 1 gallon of water?

E. 1. At the CornerMarket grocery store, you can buy eight cans of 
tomatoes for $9. The cans are the same size as those at CannedStuff, 
which sells six cans for $5. Are the tomatoes at CornerMarket a 
better buy than the tomatoes at CannedStuff?

   2. What comparison strategies did you use to choose between 
CornerMarket and CannedStuff tomatoes? Why?

Homework starts on page 40.
Applications

The problems that follow will give you practice in using rates (especially unit rates) in different situations. Be careful to use measurement units that match correctly in the rates you compute.

1. Maralah can drive her car 580 miles at a steady speed using 20 gallons of gasoline. Make a rate table showing the number of miles her car can be driven at this speed. Show 1, 2, 3, \ldots, and 10 gallons of gas.

2. Joel can drive his car 450 miles at a steady speed using 15 gallons of gasoline. Make a rate table showing the number of miles his car can be driven at this speed. Show 1, 2, 3, \ldots, and 10 gallons of gas.

3. Franky’s Trail Mix Factory gives customers the following information. Use the pattern in the table to answer the questions.

<table>
<thead>
<tr>
<th>Grams of Trail Mix</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>150</td>
<td>450</td>
</tr>
<tr>
<td>300</td>
<td>900</td>
</tr>
<tr>
<td>500</td>
<td>1,500</td>
</tr>
</tbody>
</table>

a. Fiona eats 75 grams of trail mix. How many Calories does she eat?
b. Rico eats trail mix containing 1,000 Calories. How many grams of trail mix does he eat?
c. Write an equation that you can use to find the number of Calories in any number of grams of trail mix.
d. Write an equation that you can use to find the number of grams of trail mix that will provide any given number of Calories.
For Exercises 4–8, you will explore relationships among time, rate, and distance.

4. When she drives to work, Louise travels 10 miles in about 15 minutes. Kareem travels 23 miles in about 30 minutes. Who has the faster average speed?

5. Rolanda and Mali ride bikes at a steady pace. Rolanda rides 8 miles in 32 minutes. Mali rides 2 miles in 10 minutes. Who rides faster?

6. Fasiz and Dale drive at the same speed along a road. Fasiz drives 8 kilometers in 24 minutes. How far does Dale drive in 6 minutes?

7. On a long dirt road leading to camp, buses travel only 6 miles in 10 minutes.
   a. At this speed, how long does it take the buses to travel 18 miles?
   b. At this speed, how far do the buses go in 15 minutes?

8. **Multiple Choice** Choose the fastest walker.
   A. Montel walks 3 miles in 1 hour.
   B. Jerry walks 6 miles in 2 hours.
   C. Phil walks 6 miles in 1.5 hours.
   D. Rosie walks 9 miles in 2 hours.

9. The dairy store says it takes 50 pounds of milk to make 5 pounds of cheddar cheese.
   a. Make a rate table showing the amount of milk needed to make 5, 10, 15, 20, . . . , and 50 pounds of cheddar cheese.
   b. Make a coordinate graph showing the relationship between pounds of milk and pounds of cheddar cheese. First, decide which variable should go on each axis.
   c. Write an equation relating pounds of milk \( m \) to pounds of cheddar cheese \( c \).
   d. Explain one advantage of each method (the graph, the table, and the equation) to express the relationship between milk and cheddar cheese production.
10. A dairy manager says it takes 70 pounds of milk to make 10 pounds of cottage cheese.
   a. Make a rate table for the amount of milk needed to make 10, 20, ..., and 100 pounds of cottage cheese.
   b. Make a graph showing the relationship between pounds of milk and pounds of cottage cheese. First, decide which variable should go on each axis.
   c. Write an equation relating pounds of milk \( m \) to pounds of cottage cheese \( c \).
   d. Compare the graph in part (b) to the graph in Exercise 9. Explain how they are alike and how they are different. What is the cause of the differences between the two graphs?

11. A store sells videotapes at $3.00 for a set of two tapes. You have $20. You can split a set and buy just one tape for the same price per tape as the set.
   a. How many tapes can you buy?
   b. Suppose there is a 7% sales tax on the tapes. How many can you buy? Justify your solution.

12. Study the data in these rate situations. Then write the key relationship in three ways:
   • in fraction form with a label for each part
   • as two different unit rates with a label for each rate
   a. Latanya’s 15-mile commute to work each day takes an average of 40 minutes.
   b. In a 5-minute test, one computer printer produced 90 pages of output.
   c. An advertisement for a Caribbean cruise trip promises 168 hours of fun for only $1,344.
   d. A long-distance telephone call lasts 20 minutes and costs $4.50.
Connections

Rewrite each equation, replacing the variable with a number that makes a true statement.

13. \( \frac{4}{9} \times n = 1\frac{1}{3} \)  
14. \( n \times 2.25 = 90 \)  
15. \( n \div 15 = 120 \)  
16. \( 180 \div n = 15 \)

17. Write two fractions with a product between 10 and 11.

18. Write two decimals with a product between 1 and 2.

A recent world-champion milk producer was a 4-year-old cow from Marathon, Wisconsin. The cow, Muranda Oscar Lucinda, produced a record 67,914 pounds of milk in one year! Use this information for Exercises 19–22.

19. Look back at your answers to Exercise 10. How much cottage cheese could be made from the amount of milk that Muranda Oscar Lucinda produced during her record year?

20. The average weight of a dairy cow is 1,500 pounds. How many dairy cows would be needed to equal the weight of the cottage cheese you found in Exercise 19?

21. One gallon of milk weighs about 8.7 pounds. Suppose a typical milk bucket holds about 3 gallons. About how many milk buckets would Muranda Oscar Lucinda’s average daily production of milk fill?

22. One pound of milk fills about two glasses. About how many glasses of milk could you fill with Muranda Oscar Lucinda’s average daily production of milk?

23. Some campers bike 10 miles for a nature study. Use this setting to write questions that can be answered by solving each equation. Find the answers, and explain what they tell about the bike ride.

- a. \( 10 \div 8 = \_\_\_ \)
- b. \( 1.2 \times \_\_\_ = 10 \)
- c. \( \_\_\_ \div 2 = 5 \)
The table shows the mean times that students in one seventh-grade class spend on several activities during a weekend. The data are also displayed in the stacked bar graph below the table. Use both the table and the graph for Exercises 24 and 25.

### Weekend Activities (hours)

<table>
<thead>
<tr>
<th>Category</th>
<th>Boys</th>
<th>Girls</th>
<th>All Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleeping</td>
<td>18.8</td>
<td>18.2</td>
<td>18.4</td>
</tr>
<tr>
<td>Eating</td>
<td>4.0</td>
<td>2.7</td>
<td>3.1</td>
</tr>
<tr>
<td>Recreation</td>
<td>7.8</td>
<td>6.9</td>
<td>7.2</td>
</tr>
<tr>
<td>Talking on the Phone</td>
<td>0.5</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Watching TV</td>
<td>4.2</td>
<td>3.0</td>
<td>3.4</td>
</tr>
<tr>
<td>Doing Chores and Homework</td>
<td>3.6</td>
<td>5.8</td>
<td>5.1</td>
</tr>
<tr>
<td>Other</td>
<td>9.1</td>
<td>10.7</td>
<td>10.2</td>
</tr>
</tbody>
</table>

### Weekend Activities

24. The stacked bar graph was made using the data from the table. Explain how it was constructed.

25. Suppose you are writing a report summarizing the class’s data. You have space for either the table or the graph, but not both. What is one advantage of including the table? What is one advantage of including the stacked bar graph?
26. This table shows how to convert liters to quarts.
   a. About how many liters are in 5.5 quarts?
   b. About how many quarts are in 5.5 liters?
   c. Write an equation for a rule that relates liters $L$ to quarts $Q$.

<table>
<thead>
<tr>
<th>Liters</th>
<th>Quarts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.06</td>
</tr>
<tr>
<td>4</td>
<td>4.24</td>
</tr>
<tr>
<td>5</td>
<td>5.30</td>
</tr>
<tr>
<td>9</td>
<td>9.54</td>
</tr>
</tbody>
</table>

Express each of the relationships in Exercises 27–31 as a unit rate. Label each unit rate with measurement units.

27. 12 cents for 20 beads

28. 8 cents for 10 nails

29. 405 miles on 15 gallons of gasoline

30. 3 cups of water for 2 cups of orange concentrate

31. $4 for 5 cans of soup

32. The two clocks shown below are geometrically similar. One is a reduction of the other. Each outside edge of the larger clock is 2 centimeters long. Each outside edge of the smaller clock is 1.6 centimeters long.

   a. Write an equation relating the length $L$ of any part of the large clock to the length $S$ of the corresponding part of the small clock.
   b. Write an equation relating the area $R$ of any part of the large clock to the area $M$ of the corresponding part of the small clock.
   c. Write a decimal scale factor relating lengths in the large clock to lengths in the small clock. Explain how that scale factor is like a unit rate.
Extensions

33. Chemistry students analyzed the contents of rust. They found that it is made up of iron and oxygen. Tests on samples of rust gave these data.

<table>
<thead>
<tr>
<th>Amount of Rust (g)</th>
<th>Amount of Iron (g)</th>
<th>Amount of Oxygen (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>35.0</td>
<td>15.0</td>
</tr>
<tr>
<td>100</td>
<td>70.0</td>
<td>30.0</td>
</tr>
<tr>
<td>135</td>
<td>94.5</td>
<td>40.5</td>
</tr>
<tr>
<td>150</td>
<td>105.0</td>
<td>45.0</td>
</tr>
</tbody>
</table>

a. Suppose the students analyze 400 grams of rust. How much iron and how much oxygen should they find?
b. Is the ratio of iron to oxygen the same in each sample? If so, what is it? If not, explain.
c. Is the ratio of iron to total rust the same in each sample? If so, what is it? If not, explain.

34. A cider mill owner has pressed 240 liters of apple juice. He has many sizes of containers in which to pack the juice.

a. The owner wants to package all the juice in containers of the same size. Copy and complete this table to show the number of containers of each size needed to hold the juice.

<table>
<thead>
<tr>
<th>Containers Needed by Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of Container (liters)</td>
</tr>
<tr>
<td>Number of Containers Needed</td>
</tr>
</tbody>
</table>

b. Write an equation that relates the volume $v$ of a container and the number $n$ of containers needed to hold 240 liters of juice.
In this investigation, you learned to compare rates, to find unit rates, and to use rates to make tables and graphs and to write equations. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

The Picked Today fruit stand sells three green peppers for $1.50.

1. a. Describe the process for finding a unit rate for the peppers.
   b. Find two different unit rates to express the relationship between peppers and price. Explain what each unit rate tells.
   c. Fresh Veggie sells green peppers at five for $2.25. Compare Picked Today pepper prices with Fresh Veggie prices using two different kinds of unit rates.
   d. How do you decide whether the larger unit rate or the smaller unit rate is the better buy?

2. How would you construct a rate table for green pepper prices at the two vegetable stands? Explain what the entries in the table tell.

3. a. How would you write an equation to show the price for $n$ peppers bought at Picked Today?
   b. Explain how the unit rate is used in writing the equation.
Making Sense of Proportions

In the following comparison problems, you have information about the relationship between quantities, but one or more specific values are unknown.

- **Calculators** Calculators are on sale at a price of $1,000 for 20. How many can be purchased for $1,250?

- **Similar Figures** The scale factor relating two similar figures is 2. One side of the larger figure is 10 centimeters long. How long is the corresponding side of the smaller figure?

- **Country Music** Country music is the primary format of 20% of American radio stations. There are about 10,600 radio stations in the United States. About how many stations focus on country music?

- **Doctors** Among American doctors, males outnumber females by a ratio of 15 to 4. If about 450,000 doctors are males, about how many are females?

Each of these problems can be solved in several ways. You will learn specific ways to set up ratios for problems like this and find missing values.
There are many ways to solve problems such as the ones on the previous page. One standard way is to create two ratios to represent the information in the problem. Then set these two ratios equal to each other to form a proportion. A proportion is an equation that states two ratios are equal.

For example, in the problem about doctors, you have enough information to write one ratio. Then write a proportion to find the missing quantity. There are four different ways to write a proportion representing the data in the problem.

Write the known ratio of male to female doctors. Complete the proportion with the ratio of actual numbers of doctors.

\[
\frac{15 \text{ (male)}}{4 \text{ (female)}} = \frac{450,000 \text{ males}}{x \text{ females}}
\]

Write a ratio of male to male data. Complete the proportion with female to female data.

\[
\frac{15 \text{ (male)}}{x \text{ females}} = \frac{4 \text{ (female)}}{450,000 \text{ males}}
\]

Write the known ratio of female to male doctors. Complete the proportion with the ratio of actual numbers of doctors.

\[
\frac{4 \text{ (female)}}{15 \text{ (male)}} = \frac{x \text{ females}}{450,000 \text{ males}}
\]

Write a different ratio of male to male data. Complete the proportion with female to female data.

\[
\frac{450,000 \text{ males}}{15 \text{ (male)}} = \frac{x \text{ females}}{4 \text{ (female)}}
\]

Using your knowledge of equivalent ratios, you can now find the number of female doctors from any one of these proportions.

*Does any arrangement seem easier than the others?*
Analyze the “Similar Figures” problem in the introduction.

The scale factor relating two similar figures is 2. One side of the larger figure is 10 centimeters long. How long is the corresponding side of the smaller figure?

- The scale factor means that the lengths of the sides of the larger figure are 2 times the lengths of the sides of the smaller. What is the ratio of the side lengths of the smaller figure to those of the larger figure?
- Write a proportion to represent the information in the problem.
- Solve your proportion to find the length of the corresponding side of the smaller figure.

Problem 4.1 Setting Up and Solving Proportions

A. Figure out whether each student’s thinking about each line in the following problem is correct. Explain.

Dogs outnumber cats in an area by a ratio of 9 to 8. There are 180 dogs in the area. How many cats are there?

Adrianna’s Work:

\[
\frac{9 \text{ dogs}}{8 \text{ cats}} = \frac{180 \text{ dogs}}{x \text{ cats}}
\]

1. Why did Adrianna multiply by \( \frac{20}{20} \)? How did she find what to multiply by?

2. What does this proportion tell you about the denominators? Why?

3. Is the answer correct? Explain.

Joey’s Work:

\[
\frac{8 \text{ cats}}{9 \text{ dogs}} = \frac{x \text{ cats}}{180 \text{ dogs}}
\]

4. What strategy did Joey use?

5. Why can he make this claim?
B. 1. Calculators are on sale at a price of $1,000 for 20. How many can be purchased for $1,250? Write and solve a proportion that represents the problem. Explain.

2. Country music is the primary format of 20% of American radio stations. There are about 10,600 radio stations in the United States. About how many stations focus on country music?

C. Use the reasoning you applied in Question B to solve these proportions for the variable \( x \). Explain.

1. \( \frac{8}{5} = \frac{32}{x} \)  
2. \( \frac{7}{12} = \frac{x}{9} \)  
3. \( \frac{25}{x} = \frac{5}{7} \)  
4. \( \frac{x}{3} = \frac{8}{9} \)

D. Use proportions to find the missing lengths in the following similar shapes.

1. 

2. Find the height of the tree.
In our everyday lives, we often need to solve proportion problems. So do bakers, tailors, designers, and people in many other occupations. You may have heard someone say, “A pint is a pound the world around.” This saying suggests how to compare liquid measures with weight. It tells us that a pint of liquid weighs about a pound. If you drink a quart of milk a day, you might ask, “About how much does a quart of liquid weigh?”

Problem 4.2 Applications of Proportions

A. Jogging 5 miles burns about 500 Calories. How many miles will Tanisha need to jog to burn off the 1,200-Calorie lunch she ate?

B. Tanisha jogs about 8 miles in 2 hours. How long will it take her to jog 12 miles?

C. Sam’s grandmother says that “a stitch in time saves nine.”
   1. What do you think Sam’s grandmother means?
   2. Sam’s grandmother takes 25 stitches in time. How many does she save?

D. Imani gives vitamins to her adult dogs. The recommended dosage is 2 teaspoons per day for adult dogs weighing 20 pounds. She needs to give vitamins to Bruiser, who weighs 75 pounds, and to Dust Ball, who weighs 7 pounds. What is the correct dosage for each dog?
E. The scale factor relating two similar figures is 1.8. One side of the larger figure is 12 centimeters. How long is the corresponding side of the smaller figure?

Homework starts on page 55.

4.3 Developing Strategies for Solving Proportions

When mathematicians find the same kind of problem occurring often, they look for a systematic method, or algorithm, that can be applied in each case. So far in this investigation, you have found ways to solve proportions in specific cases with nice numbers. Now you will develop general strategies that will guide you in solving proportions when the numbers are not so nicely related.

Problem 4.3 Developing Strategies for Solving Proportions

A. A jet takes 10 miles to descend 4,000 feet. How many miles does it take for the jet to descend 5,280 feet?

1. Set up two different proportions that can be solved to answer the question.
2. Solve one of your proportions by whatever method you choose.
   Check to see that your answer makes sense.

B. Jack works at a restaurant and eats one enchilada for lunch every day that he works. He figures that he ate 240 enchiladas last year. Three enchiladas have a total of 705 Calories. How many Calories did he take in last year from eating enchiladas?

1. Set up a proportion that can be solved to answer the question.
2. Solve your proportion. Check to see that your answer makes sense.
3. Describe each step in your solution strategy.
4. Can your strategy be used to solve any proportion? Explain.
5. How many Calories did he eat for lunch each working day?
C. In Pinecrest Middle School, there are 58 sixth-graders, 76 seventh-graders, and 38 eighth-graders. The school council is made up of 35 students who are chosen to represent all three grades fairly.

1. Write fractions to represent the part of the school population that is in each grade.

2. Use these fractions to write and solve proportions that will help you determine a fair number of students to represent each grade on the school council. Explain.

3. How would the number of students from each grade change if the number of members of the school council were increased to 37? Explain your reasoning.

D. Ms. Spencer needs 150 graphing calculators for her math students. Her budget allows $5,000 for calculators. She needs to know if she can buy what she needs at the discount store where calculators are on sale at 8 for $284.

She writes the following statement:

\[
\frac{8}{284} = \frac{150}{x} \quad \text{or} \quad \frac{8}{284} = 150 \div x
\]

1. Use fact-family relationships to rewrite the proportion so that it is easier to find \(x\).

2. Solve the proportion, recording and explaining each of your steps.

3. Is your method a general method that can be used to solve any proportion? Explain.

**Homework starts on page 55.**
Applications

1. Jared and Pedro walk 1 mile in about 15 minutes. They can keep up this pace for several hours.
   a. About how far do they walk in 90 minutes?
   b. About how far do they walk in 65 minutes?

2. Swimming $\frac{1}{4}$ of a mile uses about the same number of Calories as running 1 mile.
   a. Gilda ran a 26-mile marathon. About how far would her sister have to swim to use the same number of Calories Gilda used during the marathon?
   b. Juan swims 5 miles a day. About how many miles would he have to run to use the same number of Calories used during his swim?

3. After testing many samples, an electric company determined that approximately 2 of every 1,000 light bulbs on the market are defective. Americans buy more than 1 billion light bulbs every year. Estimate how many of these bulbs are defective.

4. The organizers of an environmental conference order buttons for the participants. They pay $18 for 12 dozen buttons. Write and solve proportions to answer each question. Assume that price is proportional to the size of the order.
   a. How much do 4 dozen buttons cost?
   b. How much do 50 dozen buttons cost?
   c. How many dozens can the organizers buy for $27?
   d. How many dozens can the organizers buy for $63?
5. Denzel makes 10 of his first 15 shots in a basketball free-throw contest. His success rate stays about the same for his next 100 free throws. Write and solve a proportion to answer each part. Round to the nearest whole number. Start each part with the original 10 of 15 free throws.

a. About how many free throws does Denzel make in his next 60 attempts?

b. About how many free throws does he make in his next 80 attempts?

c. About how many attempts does Denzel take to make 30 free throws?

d. About how many attempts does he take to make 45 free throws?

For Exercises 6–13, solve each equation.

6. $12.5 = 0.8x$

7. $\frac{x}{15} = \frac{20}{50}$

8. $\frac{x}{18} = 4.5$

9. $\frac{15.8}{x} = 0.7$

10. $\frac{5}{9} = \frac{12}{x}$

11. $245 = 0.25x$

12. $\frac{18}{x} = \frac{4.5}{1}$

13. $0.1 = \frac{x}{960}$

14. Multiple Choice Middletown sponsors a two-day conference for selected middle-school students to study government. There are three middle schools in Middletown.

Suppose 20 student delegates will attend the conference. Each school should be represented fairly in relation to its population. How many should be selected from each school?

A. North: 10 delegates, Central: 8 delegates, South: 2 delegates

B. North: 11 delegates, Central: 7 delegates, South: 2 delegates

C. North: 6 delegates, Central: 3 delegates, South: 2 delegates

D. North: 10 delegates, Central: 6 delegates, South: 4 delegates
Connections

For Exercises 15–17, use ratios, percents, fractions, or rates.

15. **Multiple Choice** Which cereal is the best buy?
   - **F.** a 14-ounce box for $1.98
   - **G.** a 36-ounce box for $2.59
   - **H.** a 1-ounce box for $0.15
   - **J.** a 72-ounce box for $5.25

16. Which is the better average: 10 of 15 free throws, or 8 of 10 free throws?

17. Which is the better home-run rate: two home runs per 60 times at bat, or five home runs per 120 times at bat?

18. A jar contains 150 marked beans. Scott takes several samples from the jar and gets the results shown.

   **Bean Samples**

<table>
<thead>
<tr>
<th>Number of Beans</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Marked Beans</td>
<td>3</td>
<td>12</td>
<td>13</td>
<td>17</td>
<td>27</td>
<td>38</td>
<td>52</td>
</tr>
<tr>
<td>Percent of Marked Beans</td>
<td>12%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **a.** Copy and complete the table.

   **b.** Graph the data using \((\text{number of beans}, \text{marked beans})\) as data points. Describe the pattern of data points in your graph. What does the pattern tell you about the relationship between the number of beans in a sample and the number of marked beans you can expect to find?

19. **Multiple Choice** Ayanna is making a circular spinner to be used at the school carnival. She wants the spinner to be divided so that 30% of the area is blue, 20% is red, 15% is green, and 35% is yellow. Choose the spinner that fits the description.

   - **A.**
   - **B.**
   - **C.**
   - **D.**
20. Hannah is making her own circular spinner. She makes the ratio of green to yellow 2:1, the ratio of red to yellow 3:1, and the ratio of blue to green 2:1. Make a sketch of her spinner.

21. a. Plot the points (8, 6), (8, 22), and (24, 14) on grid paper. Connect them to form a triangle.
   b. Draw the triangle you get when you apply the rule (0.5x, 0.5y) to the three points from part (a).
   c. How are lengths of corresponding sides in the triangles from parts (a) and (b) related?
   d. The area of the smaller triangle is what percent of the area of the larger triangle?
   e. The area of the larger triangle is what percent of the area of the smaller triangle?

22. The sketch shows two similar polygons.

   a. What is the length of side \(BC\)?
   b. What is the length of side \(RU\)?
   c. What is the length of side \(CD\)?
23. To earn an Explorer Scout merit badge, Yoshi and Kai have the task of measuring the width of a river. Their report includes a diagram that shows their work.

![Diagram of river measurement](image)

**a.** How do you think they came up with the lengths of the segments **AB**, **BC**, and **DE**?

**b.** How can they find the width of the river from segments **AB**, **BC**, and **DE**?

### Extensions

24. Angela, a biologist, spends summers on an island in Alaska. For several summers she studied puffins. Two summers ago, Angela captured, tagged, and released 20 puffins. This past summer, she captured 50 puffins and found that 2 of them were tagged. Using Angela’s findings, estimate the number of puffins on the island. Explain.
25. Rita wants to estimate the number of beans in a large jar. She takes out 100 beans and marks them. Then she returns them to the jar and mixes them with the unmarked beans. She then gathers some data by taking a sample of beans from the jar. Use her data to predict the number of beans in the jar.

Sample
Number of marked beans: 2
Beans in sample: 30

26. The two histograms below display information about gallons of water used per person in 24 households in a week.

**Histogram A: Water Use in 24 Households**

<table>
<thead>
<tr>
<th>Water Use Per Person (gal)</th>
<th>Number of Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>160-185</td>
<td>1</td>
</tr>
<tr>
<td>185-200</td>
<td>2</td>
</tr>
<tr>
<td>200-215</td>
<td>3</td>
</tr>
<tr>
<td>215-230</td>
<td>4</td>
</tr>
<tr>
<td>230-245</td>
<td>5</td>
</tr>
<tr>
<td>245-260</td>
<td>2</td>
</tr>
<tr>
<td>260-275</td>
<td>1</td>
</tr>
<tr>
<td>275-290</td>
<td>0</td>
</tr>
<tr>
<td>290-305</td>
<td>1</td>
</tr>
<tr>
<td>305-320</td>
<td>2</td>
</tr>
<tr>
<td>320-335</td>
<td>3</td>
</tr>
<tr>
<td>335-350</td>
<td>4</td>
</tr>
<tr>
<td>350-365</td>
<td>5</td>
</tr>
<tr>
<td>365-380</td>
<td>2</td>
</tr>
<tr>
<td>380-395</td>
<td>1</td>
</tr>
<tr>
<td>395-410</td>
<td>0</td>
</tr>
<tr>
<td>410-425</td>
<td>1</td>
</tr>
<tr>
<td>425-440</td>
<td>2</td>
</tr>
<tr>
<td>440-455</td>
<td>3</td>
</tr>
<tr>
<td>455-470</td>
<td>4</td>
</tr>
</tbody>
</table>

**Histogram B: Water Use in 24 Households**

<table>
<thead>
<tr>
<th>Water Use Per Person (gal)</th>
<th>Number of Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>160-185</td>
<td>1</td>
</tr>
<tr>
<td>185-200</td>
<td>2</td>
</tr>
<tr>
<td>200-215</td>
<td>3</td>
</tr>
<tr>
<td>215-230</td>
<td>4</td>
</tr>
<tr>
<td>230-245</td>
<td>2</td>
</tr>
<tr>
<td>245-260</td>
<td>1</td>
</tr>
<tr>
<td>260-275</td>
<td>0</td>
</tr>
<tr>
<td>275-290</td>
<td>1</td>
</tr>
<tr>
<td>290-305</td>
<td>2</td>
</tr>
<tr>
<td>305-320</td>
<td>3</td>
</tr>
<tr>
<td>320-335</td>
<td>4</td>
</tr>
<tr>
<td>335-350</td>
<td>5</td>
</tr>
<tr>
<td>350-365</td>
<td>2</td>
</tr>
<tr>
<td>365-380</td>
<td>1</td>
</tr>
<tr>
<td>380-395</td>
<td>0</td>
</tr>
<tr>
<td>395-410</td>
<td>1</td>
</tr>
<tr>
<td>410-425</td>
<td>2</td>
</tr>
<tr>
<td>425-440</td>
<td>3</td>
</tr>
<tr>
<td>440-455</td>
<td>4</td>
</tr>
</tbody>
</table>

a. Compare the two histograms and explain how they differ.
b. Where do the data seem to clump in Histograms A and B?
27. The picture at the right is drawn on a centimeter grid.

   a. On a grid made of larger squares than those shown here, draw a figure similar to this figure. What is the scale factor between the original figure and your drawing?

   b. Draw another figure similar to this one, but use a grid made of smaller squares than those shown here. What is the scale factor between the original and your drawing?

   c. Compare the perimeters and areas of the original figure and its copies in each case (enlargement and reduction of the figure). Explain how these values relate to the scale factor in each case.

28. The people of the United States are represented in Congress, which is made up of the House of Representatives and the Senate.

   a. In the House of Representatives, the number of representatives from each state varies. From what you know about Congress, how is the number of representatives from each state determined?

   b. How is the number of senators from each state determined?

   c. Compare the two methods of determining representation in Congress. What are the advantages and disadvantages of these two forms of representation for states with large populations? How about for states with small populations?
In this investigation, you used ratios and proportions to solve a variety of problems. You found that most of those problems can be expressed in proportions such as \( \frac{a}{b} = \frac{c}{x} \) or \( \frac{a}{b} = \frac{x}{c} \). The next questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. For each situation, write a problem that can be solved using a proportion. Then solve your problem.
   a. The fraction of girls in grade seven is \( \frac{3}{5} \).
   b. Bolda Cola sells at 5 for $3.
   c. Sora rides her bike at a speed of 12 miles per hour.
   d. A triangle is similar to another one with a scale factor of 1.5.

2. Write four different proportions for the problem you created in part (c). Show that the answer to the problem is the same no matter which proportion you use.

3. What procedures do you use to solve proportions such as those you wrote in Question 2?
The unit project is a mathematical investigation of a game called Paper Pool. For a pool table, use grid paper rectangles like the one shown at the right. Each outside corner is a pocket where a “ball” could “fall.”

How to Play Paper Pool

- The ball always starts at Pocket A.
- To move the ball, “hit” it as if you were playing pool.
- The ball always moves on a 45° diagonal across the grid.
- When the ball hits a side of the table, it bounces off at a 45° angle and continues to move.
- If the ball moves to a corner, it falls into the pocket at that corner.

The dotted lines on the table at the right show the ball’s path.

- The ball falls in Pocket D.
- There are five “hits,” including the starting hit and the final hit.
- The dimensions of the table are 6 by 4 (always mention the horizontal length first).
Part 1: Investigate Two Questions

Use the three Paper Pool labsheets to play the game. Try to find rules that tell you (1) the pocket where the ball will fall and (2) the number of hits along the way. Keep track of the dimensions because they may give you clues to a pattern.

Part 2: Write a Report

When you find some patterns and reach some conclusions, write a report that includes

- A list of the rules you found and an explanation of why you think they are correct
- Drawings of other grid paper tables that follow your rule
- Any tables, charts, or other tools that helped you find patterns
- Other patterns or ideas about Paper Pool

Extension Question

Can you predict the length of the ball’s path on any size Paper Pool table? Each time the ball crosses a square, the length is 1 diagonal unit. Find the length of the ball’s path in diagonal units for any set of dimensions.
The unit project is a mathematical investigation of a game called Paper Pool. For a pool table, use grid paper rectangles like the one shown at the right. Each outside corner is a pocket where a “ball” could “fall.”

The dotted lines on the table at the right show the ball’s path.

- The ball falls in Pocket D.
- There are five “hits,” including the starting hit and the final hit.
- The dimensions of the table are 6 by 4 (always mention the horizontal length first).

How to Play Paper Pool

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**Extension Question**

Can you predict the length of the ball’s path on any size Paper Pool table? Each time the ball crosses a square, the length is 1 diagonal unit. Find the length of the ball’s path in diagonal units for any set of dimensions.
The problems in this unit required you to compare measured quantities. You learned when it seems best to use subtraction, division, percents, rates, ratios, and proportions to make those comparisons. You developed a variety of strategies for writing and solving proportions. These strategies include writing equivalent ratios to scale a ratio up or down. You also learned to compute and reason with unit rates.

Use Your Understanding: Proportional Reasoning

Test your understanding of percents, rates, ratios, and proportions by solving the following problems.

1. There are 300 students in East Middle School. To plan transportation services for the new West Middle School, the school system surveyed East students. The survey asked whether students ride a bus to school or walk.
   - In Mr. Archer’s homeroom, 20 students ride the bus and 15 students walk.
   - In Ms. Brown’s homeroom, 14 students ride the bus and 9 students walk.
   - In Mr. Chavez’s homeroom, 20 students ride the bus and the ratio of bus riders to walkers is 5 to 3.

a. In what ways can you compare the number of students in Mr. Archer’s homeroom who are bus riders to the number who are walkers? Which seems to be the best comparison statement?

b. In what ways can you compare the numbers of bus riders and walkers in Ms. Brown’s homeroom to those in Mr. Archer’s homeroom? Again, which seems the best way to make the comparison?

c. How many students in Mr. Chavez’s homeroom walk to school?
d. Use the information from these three homerooms. About how many East Middle School students would you expect to walk to school? How many would you expect to ride a bus?

e. Suppose the new West Middle School will have 450 students and a ratio of bus riders to walkers that is about the same as that in East Middle School. About how many West students can be expected in each category?

2. The Purr & Woof Kennel buys food for animals that are boarded. The amounts of food eaten and the cost for food are shown below.

a. Is cat food or dog food cheaper per pound?

b. Is it cheapest to feed a cat, a small dog, or a large dog?

c. On an average day, the kennel has 20 cats, 30 small dogs, and 20 large dogs. Which will last longer: a bag of cat food or a bag of dog food?

d. How many bags of dog food will be used in the month of January? How many bags of cat food will be used?

e. The owner finds a new store that sells Bow-Chow in 15 pound bags for $6.75 per bag. How much does that store charge for 50 pounds of Bow-Chow?

f. Which is a better buy on Bow-Chow: the original source or the new store?
Explain Your Reasoning

Answering comparison questions often requires knowledge of rates, ratios, percents, and proportional reasoning. Answer the following questions about your reasoning strategies. Use the preceding problems and other examples from this unit to illustrate your ideas.

3. How do you decide when to compare numbers using ratios, rates, or percents rather than by finding the difference of the two numbers?

4. Suppose you are given information that the ratio of two quantities is 3 to 5. How can you express that relationship in other written forms?

5. Suppose that the ratio of two quantities is 24 to 18.
   a. State five other equivalent ratios in the form “p to q.”
   b. Use whole numbers to write an equivalent ratio that cannot be scaled down without using fractions or decimals.

6. What strategies can you use to solve proportions such as \( \frac{5}{8} = \frac{12}{x} \) and \( \frac{5}{8} = \frac{x}{24} \)?

7. How does proportional reasoning enter into the solution of each problem?
   a. You want to prepare enough of a recipe to serve a large crowd.
   b. You want to use the scale of a map to find the actual distance between two points in a park from their locations on the map.
   c. You want to find which package of raisins is the better value.
   d. You want to use a design drawn on a coordinate grid to make several larger copies and several smaller copies of that design.

Look Ahead

Proportional reasoning is an important way to compare measured quantities. It includes comparing numerical information by ratios, rates, and percents. It is used in geometry to enlarge and reduce figures while retaining their shapes. You will apply proportional reasoning in future Connected Mathematics units such as Filling and Wrapping, Moving Straight Ahead, and What Do You Expect?
**proportion** An equation stating that two ratios are equal. For example:

\[
\frac{\text{hours spent on homework}}{\text{hours spent in school}} = \frac{2}{7}
\]

Note that this does not necessarily imply that hours spent on homework = 2 or that hours spent in school = 7. During a week, 10 hours may have been spent on homework while 35 hours were spent in school. The proportion is still true because \( \frac{10}{35} = \frac{2}{7} \).

**tasa** Una comparación de cantidades medidas en dos unidades diferentes se llama tasa. Una tasa se puede interpretar como una comparación directa entre dos grupos (20 galletas para 5 niños) o como una cantidad promedio (4 galletas por niño). Una tasa como 5.5 millas por hora se puede escribir como \( \frac{5.5 \text{ millas}}{1 \text{ hora}} \), o como 5.5 millas a 1 hora.

**rate** A comparison of quantities measured in two different units is called a rate. A rate can be thought of as a direct comparison of two sets (20 cookies for 5 children) or as an average amount (4 cookies per child). A rate such as 5.5 miles per hour can be written as \( \frac{5.5 \text{ miles}}{1 \text{ hour}} \), or 5.5 miles : 1 hour.

**tabla de tasas** Puedes usar una tasa para hallar y organizar tasas equivalentes en una tabla de tasas. Por ejemplo, puedes usar la tasa “cinco limas por $1.00” para hacer esta tabla de tasas, en la cual se indica el número de limas y el costo de las limas.

### Cost of Limes

<table>
<thead>
<tr>
<th>Number of Limes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Limes</td>
<td>$0.20</td>
<td>$0.40</td>
<td>$0.60</td>
<td>$0.80</td>
<td>$1.00</td>
<td>$2.00</td>
<td>$3.00</td>
<td>$4.00</td>
</tr>
</tbody>
</table>
**ratio** A ratio is a number, often expressed as a fraction, used to make comparisons between two quantities. Ratios may also be expressed as equivalent decimals or percents, or given in the form $a : b$. Here are some examples of uses of ratios:

- The ratio of females to males on the swim team is $2 : 3$, or \( \frac{2}{3} \), females : males.
- The train travels at a speed of 80 miles per hour, or \( \frac{80 \text{ miles}}{1 \text{ hour}} \).
- If a small figure is enlarged by a scale factor of 2, the new figure will have an area four times its original size. The ratio of the small figure’s area to the large figure’s area will be $\frac{1}{4}$. The ratio of the large figure’s area to the small figure’s area will be $\frac{4}{1}$, or 4.
- In the example above, the ratio of the length of a side of the small figure to the length of the corresponding side of the large figure is $\frac{1}{2}$. The ratio of the length of a side of the large figure to the length of the corresponding side of the small figure is $\frac{2}{1}$, or 2.

**scale, scaling** The scale is the number used to multiply both parts of a ratio to produce an equal, but possibly more informative, ratio. A ratio can be scaled to produce a number of equivalent ratios. For example, multiplying the rate of 4.5 gallons per hour by a scale of 2 yields the rate of 9 gallons per 2 hours. Scales are also used on maps to give the relationship between a measurement on the map to the actual physical measurement.

**unit rate** A unit rate is a rate in which the second number (usually written as the denominator) is 1, or 1 of a quantity. For example, 1.9 children per family, 32 miles per gallon, and $\frac{\text{3 flavors of ice cream}}{1 \text{ banana split}}$ are unit rates. Unit rates are often found by scaling other rates.

**razón** Una razón es un número, a menudo expresado como fracción, que se usa para hacer comparaciones entre dos cantidades. Las razones también se pueden expresar como decimales equivalentes o porcentajes, o darse de la forma $a : b$. Estos son algunos ejemplos del uso de razones:

- La razón entre mujeres y hombres en el equipo de natación es $2 : 3$, es decir, $\frac{2}{3}$ mujeres : $\frac{3}{3}$ hombres.
- El tren viaja a una velocidad de 80 millas por hora, o sea, $\frac{80 \text{ millas}}{1 \text{ hora}}$.
- Si se amplía una figura pequeña por un factor de escala 2, la nueva figura tendrá un área cuatro veces mayor que su tamaño original. La razón entre el área de la figura pequeña y el área de la figura grande será $\frac{1}{4}$. La razón entre el área de la figura grande y el área de la figura pequeña será $\frac{4}{1}$, o sea, 4.
- En el ejemplo anterior, la razón entre la longitud de un lado de la figura pequeña y la longitud del lado correspondiente de la figura grande es $\frac{1}{2}$. La razón entre la longitud de un lado de la figura grande y la longitud del lado correspondiente de la figura pequeña es $\frac{2}{1}$, o sea, 2.

**tasa unitaria** Una tasa unitaria es una tasa en la que el segundo número (normalmente escrito como el denominador) es 1 ó 1 de una cantidad. Por ejemplo, 1.9 niños por familia, 32 millas por galón, y $\frac{\text{3 sabores de helado}}{1 \text{ banana split}}$ son tasas unitarias. Las tasas unitarias se calculan a menudo aplicando escalas a otras tasas.
1. In a comparison taste test of two drinks, 780 students preferred Berry Blast. Only 220 students preferred Melon Splash. Complete each statement.

a. There were ____ more people who preferred Berry Blast.

b. In the taste test, ____ % of the people preferred Berry Blast.

c. People who preferred Berry Blast outnumbered those who preferred Melon Splash by a ratio of ____ to ____.
1. Compare these four mixes for apple juice.

**Mix W**
- Concentrate: 5 cups
- Cold water: 8 cups

**Mix X**
- Concentrate: 3 cups
- Cold water: 6 cups

**Mix Y**
- Concentrate: 6 cups
- Cold water: 9 cups

**Mix Z**
- Concentrate: 3 cups
- Cold water: 5 cups

**HINT** Concentrate is the fruit substance that is left when water is removed from juice. When water is added back to the concentrate, fruit juice is made.

**a.** Which mix would make the most “appley” juice?

**HINT** What would make a juice more “appley”? Do parts (b) and (c) first to help you answer this question.

**b.** Suppose you make a single batch of each mix (W, X, Y, and Z). What fraction of each batch is concentrate?

**HINT** What is the total number of cups added to each batch?

- Mix W:
- Mix X:
- Mix Y:
- Mix Z:
c. Rewrite your answers to part (b) as percents.

Mix W: 
Mix X: 

Mix Y: 
Mix Z: 

d. Suppose you make only 1 cup of Mix W. How much water and how much concentrate do you need?

HINT For Mix W, the current batch makes 13 cups (5 cups concentrate plus 8 cups water).
What fraction of the mix is concentrate?

What fraction of the mix is water?

How can you then determine how much concentrate and water you need to get only 1 total cup (make 1 cup)?

For example, the current recipe for Mix X makes 9 cups. \( \frac{3}{9} \) (or 33 \( \frac{1}{3} \)%) of the mix is concentrate and \( \frac{6}{9} \) (or 66 \( \frac{2}{3} \)%) of the mix is water. To make one cup of Mix X, you will need the same ratio. So, you need 33 \( \frac{1}{3} \)% of the cup to be concentrate and 66 \( \frac{2}{3} \)% of the cup to be water.
1. Maralah can drive her car **580 miles** at a steady speed using **20 gallons of gasoline**. Complete the rate table showing the number of miles her car can be driven at this speed.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>116</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td><strong>580</strong></td>
</tr>
</tbody>
</table>

2. Joel can drive his car **450 miles** at a steady speed using **15 gallons of gasoline**. Complete the rate table showing the number of miles his car can be driven at this speed.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td><strong>150</strong></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
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<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td><strong>450</strong></td>
</tr>
</tbody>
</table>
2. Swimming \( \frac{1}{4} \) of a mile uses about the same number of Calories as running 1 mile.

a. Gilda ran a 26-mile marathon. About how far would her sister have to swim to use the same number of Calories Gilda used during the marathon?

**HINT** You can first fill out a table that compares the miles of swimming to the miles of running that burn the same number of Calories.

<table>
<thead>
<tr>
<th>Calories Burned</th>
<th>Miles of Swimming ( \frac{1}{4} )</th>
<th>Miles of Swimming ( \frac{2}{4} = \frac{1}{2} )</th>
<th>Miles of Swimming ( \frac{3}{4} )</th>
<th>Miles of Swimming ( \frac{4}{4} = 1 )</th>
<th>Miles of Running 1</th>
<th>Miles of Running 2</th>
<th>Miles of Running 3</th>
<th>Miles of Running 4</th>
<th>Calories Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
</tr>
</tbody>
</table>

**HINT** For every 1 mile Gilda runs, her sister only has to swim \( \frac{1}{4} \) of a mile to burn the same number of Calories (a ratio of 1 to \( \frac{1}{4} \)).

If Gilda runs 2 miles, her sister only needs to swim \( \frac{2}{4} \) of a mile (a ratio of 2 to \( \frac{2}{4} \)).
b. Juan swims 5 miles a day. About how many miles would he have to run to use the same number of Calories used during his swim?

<table>
<thead>
<tr>
<th>Miles of Swimming</th>
<th>( \frac{1}{4} )</th>
<th>( \frac{2}{4} = \frac{1}{2} )</th>
<th>( \frac{3}{4} )</th>
<th>( \frac{4}{4} = 1 )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles of Running</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calories Burned</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
<td>same amount</td>
</tr>
</tbody>
</table>

**HINT** Remember for every \( \frac{1}{4} \) mile Juan swims, he would need to run 1 mile to use the same number of Calories.